CS40 Winter 2021 Homework #3

January 26, 2021

Notes

- You may work with a partner in order to understand the problems and discuss how to approach them. If you do so, write clearly on your assignment the name of the student you collaborated with.
- Please re-read the "Conduct" section in the class syllabus.
- No late submissions! Turn-in what you have by the deadline.

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1. Use rules of inference and logical equivalences for the following (clearly state which rules you are using):
   (a) Show that the premises "Randy works hard", "If Randy works hard, then he is a dull boy", and "If Randy is a dull boy, then he will not get the job" imply the conclusion "Randy will not get the job".
   (b) Prove the implication
   $$(s \to p) \land (q \to \neg p) \to \neg(q \land s)$$
2. What are the rule of inference used in the following:
   (a) "If it snows today, the university will be closed. The university will not be closed today. Therefore, it did not snow today."
   (b) "My daughter visited France last week, therefore someone has visited France last week."
   (c) "If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material."
3. You are given the following statements:
   I All actors and journalists invited to the party are late.
   II There is at least a person who is on time.
   III There is at least an invited person who is neither a journalist nor an actor.

   Formalise the sentences using predicate logic and prove or disprove that III is a logical consequence of I and II.

4. Assume that the universe for x is all people and the universe for y is the set of all movies. Translate the statements from natural language to predicate logic using the following predicates and any needed quantifiers:

$S(x, y)$: x saw y.	$C(y)$: y is a comedy.
$L(x, y)$: x liked y.	$A(y)$: y won an award.

   (a) No comedy won an award.

   (b) Some people have seen every comedy.

   (c) No one liked every movie they have seen.

   (d) Thomas has never seen a movie that won an award.

5. Prove the following:

   (a) $x^2 = y^2$ if and only if $x = y$ or $x = -y$.

   (b) If x^3 is irrational, then x is irrational.

   (c) If x and y are even integers, then xy is even.

6. Prove that the following three statements about positive integers are equivalent (that is, each implies the other):

   I n is even;

   II $n^3 + 1$ is odd;

   III $n^2 - 1$ is odd.

7. For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first, or neither is a subset of the other.

   (a) The set of people who speak English; and the set of people who speak English with an Australian accent.

   (b) The set of fruits; and the set of citrus fruits.

   (c) The set of students studying discrete mathematics; and the set of students studying data.

8. Decide whether the following statements are true or false. You do not need to justify.

   (a) $\emptyset \in \{\emptyset\}$

   (b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$

   (c) $\{\emptyset\} \in \{\emptyset\}$

   (d) $\{\emptyset\} \in \{\{\emptyset\}\}$

   (e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$

   (f) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$

   (g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$

9. A certain company employs 100 programmers. Of these 46 can program in Java, 34 in Python, and 25 can program in both languages. How many can program in neither Python nor Java?

10. (a) Write the power set of each of the following sets:

      i. $\emptyset$

ii. $\{\emptyset\}$

iii. $\{a, \{a, b\}\}$.

(b) Let $S = \{a, b\}$.

i. List all ordered pairs (A, B) with $A \subseteq B \subseteq S$

ii. List all ordered pairs (A, B) with $A \subseteq B \subseteq S$, where B is a proper subset of S.

11. In each of the following problems, three sets are described. One of the sets is not similar to the other two. In each case, find the set that is not like the others. Give reasons for your answers.

(a) $A = \{a + b \mid a \in \mathbb{N}, b \in \mathbb{N}\}$; $B = \{a - b \mid a \in \mathbb{N}, b \in \mathbb{N}\}$; $C = \mathbb{N}$.

(b) $A = \{x^2 \mid x \in \mathbb{N}\}$; $B = \{x^2 \mid x \in \mathbb{Z}\}$; $C = \{x^2 \mid \in \mathbb{R}\}$.

(c) $A = \{4z + 2 \mid z \in \mathbb{Z}\}$; $B = \{4z - 2 \mid z \in \mathbb{Z}\}$; $C = \{2z + 4 \mid z \in \mathbb{Z}\}$.