

CS40 Winter 2021 Homework #1

January 5, 2021

Notes

- You may work with a partner in order to understand the problems and discuss how to approach them. If you do so, write clearly on your assignment the name of the student you collaborated with.
- Please re-read the “Conduct” section in the class syllabus.
- No late submissions! Turn-in what you have by the deadline.

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1. State whether or not each sentence is a proposition. If it is a proposition, what is its truth value?
 - (a) Paris is the capital of Denmark.
 - (b) Don't eat the daisies.
 - (c) Take two aspirin and call me in the morning.
 - (d) $x + 3 = 7$ and $x = 4$
 - (e) $2 + 7 = 9$
 - (f) $2 - 7 = 5$
 - (g) This statement is false.
 2. Write the negation of each of these propositions:
 - (a) George Washington was the first president of the United States.
 - (b) $1 + 5 = 7$
 - (c) It is hot today.
 - (d) 6 is negative.
 3. Is the use of “or” in these English sentences intended to be inclusive or exclusive?
 - (a) “If you fail to make a payment on time or fail to pay the amount due, you will incur a penalty.”
 - (b) “If I can't schedule the airline flight or if I can't get a hotel room, then I can't go on the trip.”
 - (c) “She has one or two brothers.”

- (d) “If you do not wear a shirt or do not wear shoes, then you will be denied service in the restaurant.”
4. A sign at the entrance of a restaurant declares: “No shoes, no shirt, no service.” Write this sentence as a conditional proposition.
5. Write the negation of the statement “If you pay your membership dues, then if you come to the club, you can enter free”.
6. Write the contrapositive, converse, and inverse of the following propositions:
- (a) “If the number is positive, then its square is positive.”
- (b) “I stay home whenever it is stormy.”
7. Let p stand for the proposition “I did well in CS40”, and q for “I understand discrete mathematics”. Express the following as natural English sentences:
- (a) $\neg p$
- (b) $p \vee q$
- (c) $p \wedge q$
- (d) $p \rightarrow q$
- (e) $\neg p \rightarrow \neg q$
- (f) $\neg p \vee (p \wedge q)$
8. Suppose u represents “you understand the material”, s represents “you study the theory”, and w represents “you work on exercises”. Write the following compound proposition using u , s , w and the appropriate connectives: “You study the theory and work on exercises, but you don’t understand the material”.
9. Show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent.
10. Use a truth table to verify:
- (a) The first De Morgan’s Law: $\neg(p \wedge q) \equiv \neg p \vee \neg q$
- (b) The second De Morgan’s Law: $\neg(p \vee q) \equiv \neg p \wedge \neg q$
11. Show that each of these conditional statements is a tautology by using truth tables.
- (a) $[\neg p \wedge (p \vee q)] \rightarrow q$
- (b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
- (c) $[p \wedge (p \rightarrow q)] \rightarrow q$
- (d) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$
12. Solve the following logic puzzles:
- (a) You come across two people. P says “I am lying if Q is,” and Q says, “ P is lying if I am.” Can you tell who if anyone is telling the truth?
- (b) You come across three people. P says, “If Q is lying, then so is R ,” Q says, “If R is lying, then so is P ,” and R says, “If P is lying, then so is Q .” Who if anyone is telling the truth?